

Vanishing Hull

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Abstract

Vanishing points are valuable in many vision tasks such as orientation estimation, pose recovery and 3D reconstruction from a single image. Many methods have been proposed to address the problem, however, a consistent framework to quantitatively analyze the stability and accuracy of vanishing point estimation is still absent. This paper proposes a new concept, vanishing hull, which solves the problem.

Given an edge error model, the range of a true edge can be modeled using a fan region. The intersection of all these fan regions is a convex hull, which is called vanishing hull. A vanishing hull gives the region of a true vanishing point, and its distribution determines the probability of the vanishing point. The expectation of the vanishing hull is the optimal solution of the vanishing point, its variance defines the accuracy of the estimation, and its shape determines the stability of the vanishing point. Hence, we can quantitatively analyze the stability and accuracy of the vanishing point estimation using vanishing hull. Simulation results show that our method is significantly better than one state-of-the-art technique, and real data results are also promising.

1. Introduction

A vanishing point is defined as the intersection point of a group of image lines that correspond to the projection of parallel lines in 3D with an ideal pin-hole camera model. The position of a vanishing point in the image plane is only determined by the camera center and the orientation of the 3D lines in the camera system. Vanishing points are valuable in many vision tasks, including buildings detection in aerial images[15], camera poses recovery [5], robots navigation [14] and 3D reconstruction from a single image [9].

Many research have been conducted in accurately identifying the position of vanishing points. This varies from simple line grouping [8] to more complicated methods using a statistical model [6]. The two key problems in identifying vanishing points are finding the group of image lines that correspond to a true vanishing point and com-

puting the position of the vanishing point with the presence of image noises. Most previous research work [1] [12] focus on the first problem, while the computing of vanishing points are accomplished using the least square technique or searching for the maxima vote in the Gaussian sphere [4]. The performances are often evaluated empirically, furthermore, few work have been conducted on finding a theory to quantitatively analyze the stability and accuracy of vanishing points estimation with image noises.

The image lines that correspond to a vanishing point can be grouped using clustering methods or voting methods. Clustering methods first find possible clusters using the intersection of all pairs of lines [10] or image gradient orientations [12], then assign each line to different clusters using a distance or angle criterion. The drawbacks of clustering methods are the high computational complexity and that a hard threshold is needed to group lines into clusters. Voting methods include image space methods [13] and Gaussian sphere methods [4]. Hough Transform in Gaussian sphere space is a global feature extraction method, hence sensitive to spurious maxima. The knowledge of primitive models can be used to reduce spurious maxima [15]. However, the accuracy of Gaussian sphere methods is limited to the discretization of the accumulator space, hence it is hard to achieve the precision that an image can provide.

For computing the vanishing points with image noises, the least square method is generally used [2]. Several improvements have been made to the least square method. McLena and Kotturi [12] integrate edge detection and line clustering to the process of vanishing points detection, and then use a non-linear method to compute the position of vanishing points with a statistical edge error model. Shufelt [15] uses a fan edge error model to improve the robustness to image noises in the Gaussian sphere. However, the performances of both methods are evaluated empirically. Liebowitz and Zisserman [10] use a Maximum-Likelihood estimator to compute the position of the vanishing point, however, the performance is only visually analyzed. In general, a consistent framework to quantitatively analyze the computation of vanishing points with image noises is still absent, which is the main impetus of this work.

Observing that the region of an edge error model with images noises is a fan region (Figure 1), we intersect all the

fan edges to form a convex polygon, called the *Vanishing Hull*. This paper shows that the vanishing hull gives the region of the true vanishing point, and its distribution gives the probability of the vanishing point. The expectation of the vanishing hull, the centroid for a uniform distribution, gives the optimal solution of the vanishing point under statistical meanings, its variance defines the accuracy of the vanishing point and its shape determines the stability of the vanishing point. Hence, the vanishing hull concept provides a theoretical framework to quantitatively analyze the region, optimal solution, stability and accuracy of vanishing points. Besides a framework for analyzing vanishing points with image noises, we also present a novel edge grouping method based on heuristic filters without any hard thresholds.

Since the idea of vanishing hull is derived from the intersection of edge regions, we first present a simple edge error model (Section 2.1) and the corresponding edge grouping method (Section 2.2). The vanishing hull concept is first introduced based on the simple edge error model (Section 2.3), and then extended to general edge error models (Section 2.4). The performance of our method is quantitatively compared with one state-of-the-art technique [10](Section 3), and we conclude the paper in Section 4.

2. Vanishing hull

2.1. Edge error model

Different edge error models have been presented. McLean and Kotturi [12] use a statistic model to present both the error of the line centroid and orientation. Shufelt [15] presents a simple but effective edge error model. Inspired by the idea of a fan edge region [15], we derive the concept of vanishing hull by the intersection of all these regions. We first adapt this simple edge error model to our vanishing hull framework, and then extend the concept to general edge error models.

Consider the representation of a line segment using two endpoints, and assume the two end points have one pixel precision, then two fan regions with a rectangular region in the middle can be formed by moving the two end points freely in the pixel squares (Figure 1). This region is a concave region, so we cannot guarantee that the intersection of such regions will be convex. Fortunately, a true vanishing point cannot lie in the image line segment (Section 2.2), so the rectangular region has no effect on the shape of the intersection of edge regions. We simply take the middle point of the edge, and form two fan regions with the two end points. Furthermore, a true vanishing point

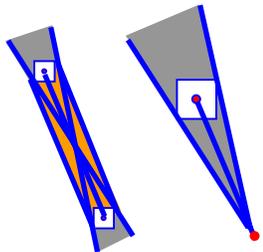


Figure 1. Edge error model.

can only lie in one direction of the edge, so we can just take one of the fan region, which is a convex region (Figure 1).

2.2. Line clustering

Since we are interested in finding the intersection regions of all the edges, it is necessary to first identify the image lines that can form a possible vanishing point, i.e., we need to group lines into different clusters. We opt to use the clustering method in image space rather than Gaussian sphere because of several reasons. First, Gaussian sphere method is a global feature extraction method, which is sensitive to spurious maxima. Second, the accuracy is limited to the discretizing accuracy, hence hard to achieve the precision that an image can offer. The last and most important reason, the intersection of the fan regions of the edges that belong to the maximum cell in the Gaussian sphere may be empty, which makes the vanishing hull meaningless.

Finding clusters

The Canny edge detector is used to find sub-pixel edges, then Hough Transform is used to find possible lines, and nearby lines are connected using some user defined threshold. The lines are represented using two end points.

The intersections of all pairs of line segments are computed to find all possible clusters. The computational complexity is $O(n^2)$, where n is the number of lines. Grouping lines into different clusters takes $O(n)$ time, so the overall time complexity is $O(n^3)$, which is expensive for a large number of lines. We will reduce the complexity using a filtering step and the RANSAC algorithm later.

Grouping

After finding the clusters, we need a criterion to assign lines to different clusters. A distance criterion gives priority to close vanishing points, while an angle criterion gives priority to far vanishing points. A reasonable threshold is to use a tuple of both distance and angle, or use normalized angle error. However, all these methods need a hard threshold, which may be inconsistent with the edge error model.

We use a method that is consistent with the edge error model without any hard threshold. For each cluster of two lines, we can find the intersection region A of the edges, and a test edge is assigned to this cluster when its edge region overlaps with region A . Furthermore, we use a strong constraint for clustering (Figure 2). An edge is assigned to a cluster only if its edge region covers the intersection point of the cluster. This guarantees that the intersection region of the edge regions in each

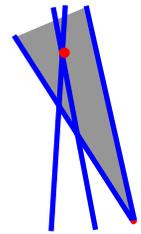


Figure 2. Grouping with an edge error model.

cluster is not empty. The normalized length of each edge is accumulated in its assigned cluster, and the maximum clusters are chosen to compute potential vanishing points.

Filtering spurious vanishing points

Most of our testing images are outdoor building images with heavy occlusion by trees (Figure 5), which causes many spurious vanishing points. Knowledge of the image and vanishing points are used to filter spurious vanishing points. First we roughly classify the extracted lines into x and y groups according to the line orientation, which dramatically reduce the size of line number. Then vanishing points are filtered using the following three filters.

1) Iterative line length. According to the edge error model, longer lines are more reliable, however, we would like also to keep shorter lines. So we first filter the lines using large length threshold, then estimate the possible vanishing points, and these points are used to find more line supporters according to the grouping method.

2) Covering area. Another observation of the image is that edges of trees only cover a small part of the image region, so the ratio of the covering area against the image area is also used to filter spurious vanishing points.

3) Valid vanishing point. Vanishing points are the intersection of image lines that correspond to parallel lines in 3D. So by definition, a valid vanishing point will not lie on the image segment in the image space, which is very effective in reducing spurious clusters.

RANSAC

Even though we classify lines into two directions to reduce the line number, and use filters to reject spurious clusters, the number of clusters may still be large. Since we are interested in find vanishing points that correspond to dominant directions, the RANSAC algorithm is used to find the maximum cluster of lines for x - y direction. The vanishing point of z direction is estimated using the orthogonal property of the three directions, and its supporting lines are found using our grouping method to refine the position using vanishing hull.

2.3. Vanishing hull

Definition and property

A *vanishing hull* is defined as the intersection of the fan-shape edge regions with a given edge error model. Figure 5. (d) shows a vanishing hull of a real image. A vanishing hull has the following properties.

Property I. A vanishing hull is not empty. *Proof:* according to the grouping method, an edge is assigned to a cluster only if its edge region covers the intersection point of the cluster, so the vanishing hull contains at least the intersection point.

Property II. A vanishing hull is convex. *Proof:* the intersection of convex regions (edge regions are fan shape) is convex.

Property III. A true vanishing point lies inside the region of its vanishing hull with the assumption of correct edge error model (the true edge lies inside the edge fan). *Proof:* By definition, the true vanishing point must lie inside the union of all the edge regions. Now assuming the vanishing point VP lies outside of the vanishing hull but inside the union of the edge region. Then, there must exist some edge, say L , whose edge region does not cover VP . Hence the edge error model of edge L is wrong, which is contradictory with our assumption. Hence, VP must be inside the vanishing hull. This property is important, it tells us where to find the true vanishing point.

Property IV. The centroid of a vanishing hull is the optimal estimation of the vanishing point with a uniform distribution model. *Proof:* the optimal estimation of the vanishing point is the expectation of the probability distribution of the vanishing points inside the vanishing hull under statistic meaning. With a uniform distribution, the expectation of a vanishing hull is its centroid.

Property V. The variance of a vanishing hull determines the accuracy of the estimated vanishing point. *Proof:* this is directly from probability theory.

Property VI. The shape of a vanishing hull determines the stability of the estimation of the vanishing point.

A vanishing hull can be open, it can also be a closed non-trivial convex polygon, a line segment or a point. When the image lines are parallel, the vanishing hull is an open convex hull; the centroid is undetermined, which means the estimation of the vanishing point is unstable. This is reasonable because edges have noise, any non-zero noise will be enlarged to infinity when the vanishing point is at infinity, which makes the estimation unreliable. An open vanishing hull indicates a vanishing point at infinity, which corresponds to a point at the great circle parallel to the image plane in the Gaussian sphere. We handle this case by setting the vanishing point to infinity.

When the vanishing hull is a non-trivial convex polygon, the vanishing point can be reliably estimated using the centroid with the variance of the distribution as the estimation accuracy. When the vanishing hull shrinks to a line segment, the uncertainty is along just one direction, and the vanishing point can be precisely computed when the vanishing hull degenerates to a point, which corresponds to an error-free edge model.

Determining vanishing hull

Now let's consider how to find the vanishing hull. A fan shape edge region can be considered as the intersection of two half planes (Figure 1), so the problem of intersection of the edge regions can be cast as the problem of half-planes intersection. A naive way to solve the problem is to add one half-plane bound line a time, and compute the intersection region, which takes $O(n^2)$ time. However, a more elegant algorithm with $O(n \lg n)$ time can be presented by

study the property of dual space [3].

There exists an interesting property called duality between lines and points [3]. Given a non-vertical line L , it can be expressed using two parameters (k, b) . We can define a point with k as the x coordinate and b as the y coordinate. This point is called the dual point (L^*) of line L . The space of the lines is called the prime space, while the space of the points is called the dual space. The half-plane intersection problem in the prime space can be mapped as the problem of finding the convex hull of the points in the dual space.

We first divide the bound lines of half-planes into two sets: an upper set (half-planes lie above the bound lines) and a lower set (half-planes lie below the bound lines). Let's consider the upper set first. For each line L in the upper set, we can find a dual point $L^*(k, b)$. The intersection of all the half-planes in the upper set can be mapped as finding the lower convex hull (the edges of the lower boundary of the convex hull) of the dual points. The proof is out of the scope of this paper, readers may refer to [3]. Similarly, we can find the intersection of all the half-planes in the lower set by determining the upper convex hull of the dual points. Notice that according to our dual mapping, the upper convex hull and the lower convex hull will not intersect although the upper and lower half-planes do intersect, and that is the reason we split the whole half planes into two sets. Finally, the two regions are merged to find the vanishing hull. The algorithm is summarized as following:

1. Split the half plane into two sets, an upper set and a lower set.
2. Map each set into a dual space and find the corresponding upper and lower convex hull (notice the two convex hulls are different).
3. Map the two half convex hulls back to prime space, and merge them to find the vanishing hull.

Finding a convex hull of a point sets is a well-defined problem [3], which takes $O(n \lg n)$, with n as the number of points. The mapping and merging takes linear time, so the overall time complexity is $O(n \lg n)$.

Expectation and variance of vanishing hull

With a uniform probability distribution, the expectation and variance of the position of a true vanishing point can be computed using Equation (1), where D is the region of the vanishing hull, and A is its area. We only show x coordinate due to lack of space, y coordinate can be computed in a similar way. Given a list of vertices of the vanishing hull, it is easy to show that the mean can be computed using the coordinates of these vertices (Equation 2, 3).

$$\mu(x) = \frac{1}{A} \iint_D x dA \quad \text{var}(x) = \frac{1}{A} \iint_D (x - \mu(x))^2 dA \quad (1)$$

$$A = \frac{1}{2} \sum_{i=0}^{n-1} (x_i y_{i+1} - x_{i+1} y_i) \quad (2)$$

$$\mu(x) = \frac{1}{6A} \sum_{i=0}^{n-1} (x_i + x_{i+1})(x_i y_{i+1} - x_{i+1} y_i) \quad (3)$$

The variance of the vanishing hull can also be represented as a simple expression of the coordinates of the vertices. According to Green's calculus theorem [16], an integral in the region can be converted to an integral on the boundary:

$$\int_{\partial D} f(x, y) dx + g(x, y) dy = \iint_D \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y} \right) dx dy \quad (4)$$

Let $f = 1$, $g = \frac{1}{3}(x - \mu(x))^3$, we have

$$\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y} = (x - \mu(x))^2 \quad (5)$$

$$\begin{aligned} \text{So: var}(x) &= \frac{1}{A} \iint_D (x - \mu(x))^2 dA = \frac{1}{A} \iint_D \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y} \right) dA \\ &= \frac{1}{A} \int_{\partial D} f(x, y) dx + g(x, y) dy \end{aligned} \quad (6)$$

Let's consider the integral on line (x_i, y_i) to (x_{i+1}, y_{i+1}) , we can parameterize the point on the line using t , such that:

$$x = x_i + t(x_{i+1} - x_i) \quad y = y_i + t(y_{i+1} - y_i) \quad t \in [0, 1] \quad (7)$$

Let $a = (x_i - \mu(x))$, $b = (x_{i+1} - x_i)$, $c = (y_{i+1} - y_i)$, it is easy to show that the variance of x coordinate can be computed as:

$$\text{var}(x) = \frac{1}{3} \sum_{i=0}^{n-1} c(a^3 + \frac{3}{2}a^2b + ab^2 + \frac{b^3}{4}) \quad (8)$$

Similarly, we can compute the variance of y coordinate.

2.4. Vanishing hull for general edge error model

The vanishing hull concept can be easily extended to general statistical edge error models. First we show an augmented vanishing hull considering the full edge region, and then we show that a general vanishing hull can be derived in a similar way.

In Section 2.1 we ignored the rectangular region, and use one of the edge fan to derive the vanishing hull concept. We claim that the shape of the vanishing hull will not change with this approximation, which is true. However, the probability distribution in the vanishing hull is not a uniform distribution. For a full edge span region, an angle (called extreme angle θ) is formed by the vanishing point VP and two extreme points P_1 and P_2 (Figure 3). According to the edge error model, the two end points have equal probability inside the one-pixel-size square. The probability of a true edge passing through the vanishing point VP is determined by the overlapping area of the extreme angle and the pixel squares.

Let $p_i(l_i, VP)$ be the probability of a true edge l_i pass-

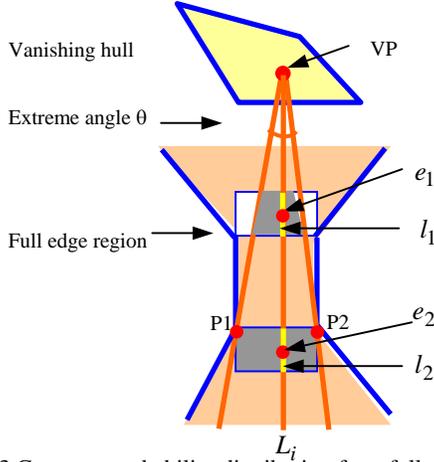


Figure 3. Compute probability distribution for a full edge region.

ing through VP , e_1 and e_2 be the two end points of l_i , and S_1 and S_2 be the overlapping region of the extreme angle with the two pixel squares, then:

$$p_i(l_i, VP) = p(e_1 \in S_1 \& e_2 \in S_2) \quad (9)$$

Assuming the two end points are independent with probability density function (PDF) $f(x, y)$ and $g(x', y')$ respectively, then the joint PDF is $f(x, y) * g(x', y')$, which is a 4D uniform distribution. Then $p_i(l_i, VP)$ is the integral of the joint PDF over the region $S_1 S_2$.

$$p_i(l_i, VP) = \iint_{e_1 \in S_1 \& e_2 \in S_2} f(x, y) * g(x', y') dS_1 dS_2 \quad (10)$$

We can use a line-sweeping method to compute the integral in a region. Now consider a sweep line L_i that passes VP , and intersects the two squares with line segment l_1 and l_2 (Notice L_i is a line inside the extreme angle, while l_i is a true edge inside the whole edge region). Line L_i sweeps the whole overlapping region when its angle varies from 0 to θ , the integral of the joint PDF over the two line segments is:

$$p(e_1 \in l_1 \& e_2 \in l_2) = \int_{e_1 \in l_1 \& e_2 \in l_2} f(x, y) * g(x', y') dl_1 dl_2 \quad (11)$$

Denote the end points of line segment l_1 as $(x_1, y_1) (x_2, y_2)$ and l_2 as $(x_1', y_1') (x_2', y_2')$, they form a line segment in a 4D space (because we have three linear constraints: l_1 , l_2 and they have the same slope). The integral of a uniform distribution over a line segment in 4D is just the length of the line segment, hence:

$$p(e_1 \in l_1 \& e_2 \in l_2) = [(x_2 - x_1)^2 + (y_2 - y_1)^2 + (x_2' - x_1')^2 + (y_2' - y_1')^2]^{\frac{1}{2}} \quad (12)$$

Now we can integrate over the angle to get the region integral:

$$p_i(l_i, VP) = \int_{\theta} p(e_1 \in l_1(\theta) \& e_2 \in l_2(\theta)) d\theta \quad (13)$$

The full expression of the analytical PDF of the vanishing hull for a full edge region is complicated, and the distribution function may not be continuous over the entire region. Even when the distribution for a single edge is continuous, the overall PDF is very high order due to the large number of edges. An analytical solution to the integral of such a high order non-linear PDF is very complicated, and may not exist. We use a discretizing method to solve this problem. The discretization process can achieve high precision (one pixel) because the vanishing hull region is bounded. Consider the center point, $VP(x, y)$, of a cell with one pixel size, we can find the extreme angle relative to edge l_i , and then compute the probability $p_i(l_i, VP(x, y))$ according to Equation 13. Then the probability of the point $VP(x, y)$ over all the edges is computed and normalized (Equation 14). The expectation and variance can be easily computed in the discretized vanishing hull.

$$P(x, y) = \prod_{i=0}^{n-1} p_i(l_i, VP(x, y)) \quad P^*(x, y) = \frac{P(x, y)}{\sum P(x, y)} \quad (14)$$

Such a vanishing hull considering the full edge span region is called ‘‘augmented vanishing hull’’. In practice, we found that the vanishing hull often consists of only a few vertices (less than 10 vertices for 1000 lines), which means that the probability of a vanishing point close to the edge region’s boundary is very low. Since vanishing points close to the middle of the edge region has similar overlapping area with the pixel squares, it is reasonable to assume a uniform distribution for the vanishing hull formed of full edge regions.

We can extend the vanishing hull concept to general edge error model in a way similar to augmented vanishing hull. A general edge error model often models the error of the edge centroid and orientation [12], or the two end points [15], using a Gaussian distribution. The edge span region is still a fan shape, so the intersection of the edge regions is a convex hull. Assuming the PDF of the line l_i passing a vanishing point (x, y) is $f_i(x, y)$, then the PDF of the vanishing hull over all the lines is $\prod_i f_i(x, y)$. Again, this

function is a high order non-linear function, we discretize the vanishing hull, and compute the mean and variance of the vanishing hull.

3. Analysis and result

3.1. Simulation data

We first analyze the theory using synthetic data. The goal of the simulation is to show that a vanishing hull gives the region of the true vanishing point, its expectation is the

optimal solution, its shape determines the stability and its variance determines the accuracy of the estimation. The simulation is designed as following. A group of 3D parallel lines are projected by an ideal pin-hole camera to an image plane, then random noises with specified magnitude are added to the end points. The vanishing point is estimated using the centroid of the vanishing hull assuming a uniform

Table 1. Parameter settings.

Parameter	Range	Other parameter settings
1.line orientation angle (degree)	$\theta \in [0.01, 40]$	$f_{ov} = 40$ $l = 50$ $\varepsilon = 0.5$ $n = 200$
2.camera field of view (degree)	$f_{ov} \in [20, 80]$	$\theta = 1$ $l = 50$ $\varepsilon = 0.5$ $n = 200$
3.image line length (pixel)	$l \in [10, 100]$	$\theta = 10$ $f_{ov} = 40$ $\varepsilon = 0.5$ $n = 200$
4.image noise magnitude (pixel)	$\varepsilon \in [0.05, 0.5]$	$\theta = 1$ $f_{ov} = 40$ $l = 50$ $n = 200$
5.number of image lines	$n \in [20, 1000]$	$\theta = 5$ $f_{ov} = 40$ $l = 50$ $\varepsilon = 0.5$

distribution. We extensively analyze our Vanishing Hull (VH) algorithm with different parameter settings (Table 1). For each of the five groups of parameter settings, we sample the space with 100 evenly distributed intervals, the other four parameters are set as constant when one parameter varies in its range to test the performance relative to each parameter.

Vanishing hull is the true vanishing point region

The simulation shows that all the true vanishing points lie inside the vanishing hull. This is logical because the maximum noise magnitude is specified, so the edge error model exactly predicts the region of the true edge region, hence the true vanishing points lie inside the vanishing hull.

Expectation is the optimal solution

The result of VH method is compared with other two methods, Least Square (LS) method and Maximum Likelihood (ML) method [10], to show that the expectation is the optimal solution. The comparison criterion is the recovered orientation angle error relative to the ground truth.

LS method uses least square to find the vanishing point closest to all lines, and ML method uses a non-linear method to minimize the distance of the line that passes the vanishing point and the mid-point to the two end points. According to our implementation, the difference of the ML and LS method is often several pixels, so the angle difference is very small. This is because ML method uses a non-linear optimization method, which often gives a local minimum close to the result of LS method. We just show the result of VH and ML method to make the figure clear (Figure 4).

The parameter θ and f_{ov} are related to the perspective

effect of images. The result (Figure 4. (a), (b)) shows that the ML method gives large errors for weak perspective images (up to 40 degrees when the orientation angle is less than 0.1 degrees), while VH method performs reliably with maximum angle error less than 0.5 degrees and average less than 0.1 degrees. When the perspective effect is strong (orientation angle is larger than 10 degrees), the vanishing hull shrinks to several dozens of pixels, both methods perform well.

The parameter l and ε are related to the quality of edges. The simulation (Figure 4. (c), (d)) shows that the ML method gives large orientation errors for poor quality edges ($l < 30$ or $\varepsilon > 0.2$), and the maximum angle error is more than 14 degrees. The VH method performs significantly better than the ML method and very reliable over the whole range (maximum angle error 0.3 degrees, and average angle error less than 0.1 degrees).

The last parameter (Figure 4. (e)) compares the performance against the number of image lines. The simulation

shows that the number of lines has no strong effects on the performance of ML method, however, it is a critical parameter for the performance of the VH method. When the number is more than 500, VH method performs very reliably, when the number drops to 100, it still performs significantly better than the ML method. However, when the number of lines drops below 50, the result is mixed. There are several reasons for this. First, the region of a vanishing hull shrinks with the increasing number of lines, so it is more reliable for more lines. Second, the expectation of the vanishing hull is the optimal solution for a vanishing point under statistical meaning. However, when the sample (number of lines) is small, the true value of the vanishing point may deviate from the statistical value. The last reason is that we use a uniform distribution, which is an approximation as we showed in section 2.4.

In general, the VH method performs significantly better than the other two methods, especially for weak perspective images, and the performance of the VH method is very reliable with several hundreds of lines, a reasonable number for high-resolution images of buildings and aerial images. This shows that the expectation of the vanishing hull is the optimal solution of vanishing points estimation.

Stability and accuracy

We can also predict unstable vanishing points using the VH method. For most of the cases, the vanishing hull is closed, which indicates that the vanishing point is stable. When the vanishing hull is open, it indicates that the vanishing point is at infinity. We visualize unstable vanishing points with blue color in Figure 4.(f), where $\theta = 0.01$. Notice that the VH method achieves an average error less than 0.5 degrees even for such an ill-conditioned case. The error of the VH method is also within the magnitude of the

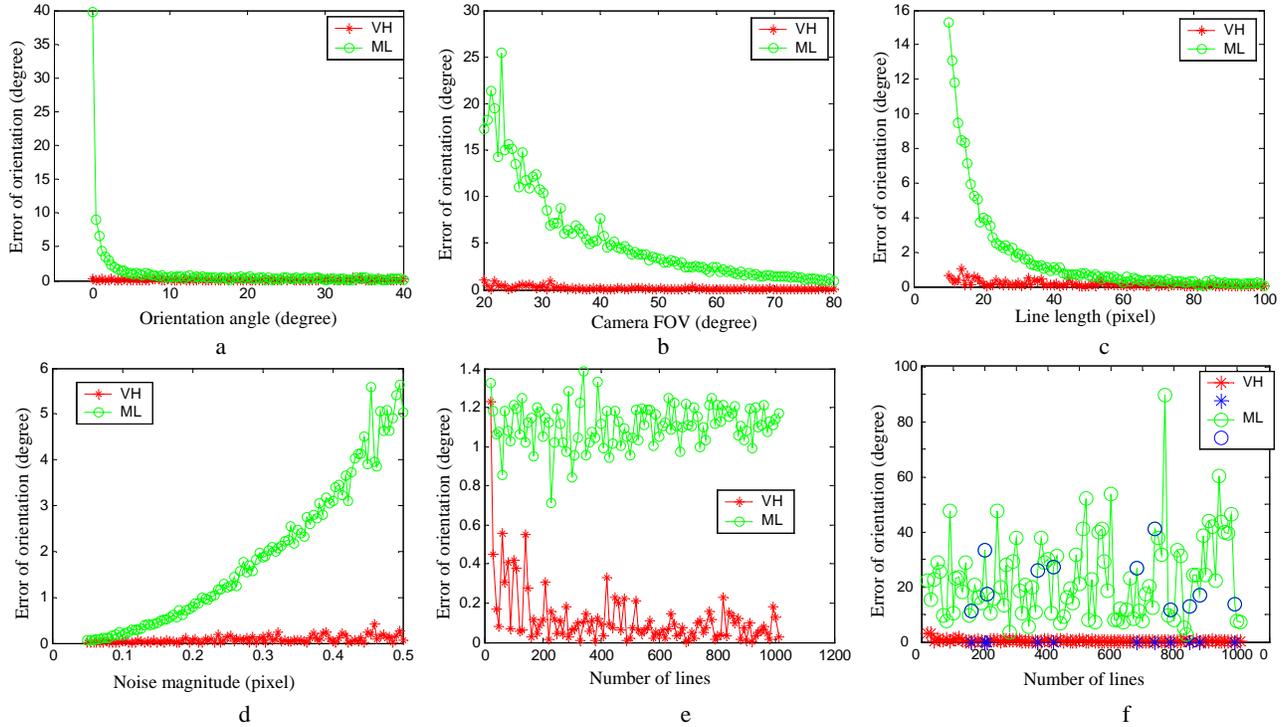


Figure 4. Performance comparison of VH and ML method on synthetic data.

variance for all five groups of parameters, which implies that the variance determines the accuracy of the estimation. We did not show the graph of the variance due to limited space.

3.2. Real data

The vanishing hull theory is also tested using real data. Since the concept of the vanishing hull is derived from the edge error model, its property (optimal estimation, stability and accuracy) depends on the edge error model, which makes it a valuable tool to analyze the performance of different edge error models.

We tested the algorithm with two images. The camera is calibrated, and lens distortion is corrected. The noise magnitude of the end points of edges is modeled as: $\varepsilon = \frac{c}{\sqrt{l}}$,

where l is the length of the edge, and c is set to 3.5 pixels. This model shows a better result than setting the noise magnitude as a constant value for all lines.

The first image is a strong perspective image occluded by heavy trees. Figure 5. (a), (b) compares the result before and after filtering vanishing points, spurious vanishing points caused by trees and small-scale textures are filtered using our method, which shows the effectiveness of our filter. The vanishing hull (Figure 5. (d)) is found using dual-space algorithm, the vanishing point is computed as the centroid assuming a uniform distribution, and the image is rectified (Figure 5. (c)) [10]. It is hard to verify the property of the vanishing hull for a real image due to lack of ground truth, so we opt to use a manual verification by

measuring the horizontal and vertical line angle error. Careful manual user verification shows that the horizontal angle error is less than 0.1 degrees (the standard deviation of the vanishing hull is $\sigma=0.12$ degrees), and vertical angle less than 0.15 degrees ($\sigma=0.17$), while ML method gives 0.2 and 0.3 degrees angle error respectively.

The second image is a weak perspective image (Figure 5. (e)), which is a very challenging example for vanishing point algorithms. The result of VH method shows that x direction vanishing point is 2×10^6 , and y direction is -4×10^4 . Careful manual user verification shows that the horizontal angle error of VH method is less than 0.3 degrees ($\sigma=0.35$), and vertical angle less than 0.2 degrees ($\sigma=0.22$), while ML method gives 1.94 and 0.55 degrees angle error respectively.

Both images show that the VH method gives better performance (optimal solution), and the user verified orientation error is within the range of variance (accuracy). Hence the true vanishing points are within the region of the vanishing hull. All the vanishing points are stable because the vanishing hulls are all closed convex polygons.

4. Conclusion

Vanishing points are valuable in many vision tasks such as orientation and pose estimation. This paper defines the concept of vanishing hull, which is the intersection of the edge regions. The vanishing hull gives the region of the true vanishing point, and its probability distribution determines the property of vanishing points. The expectation

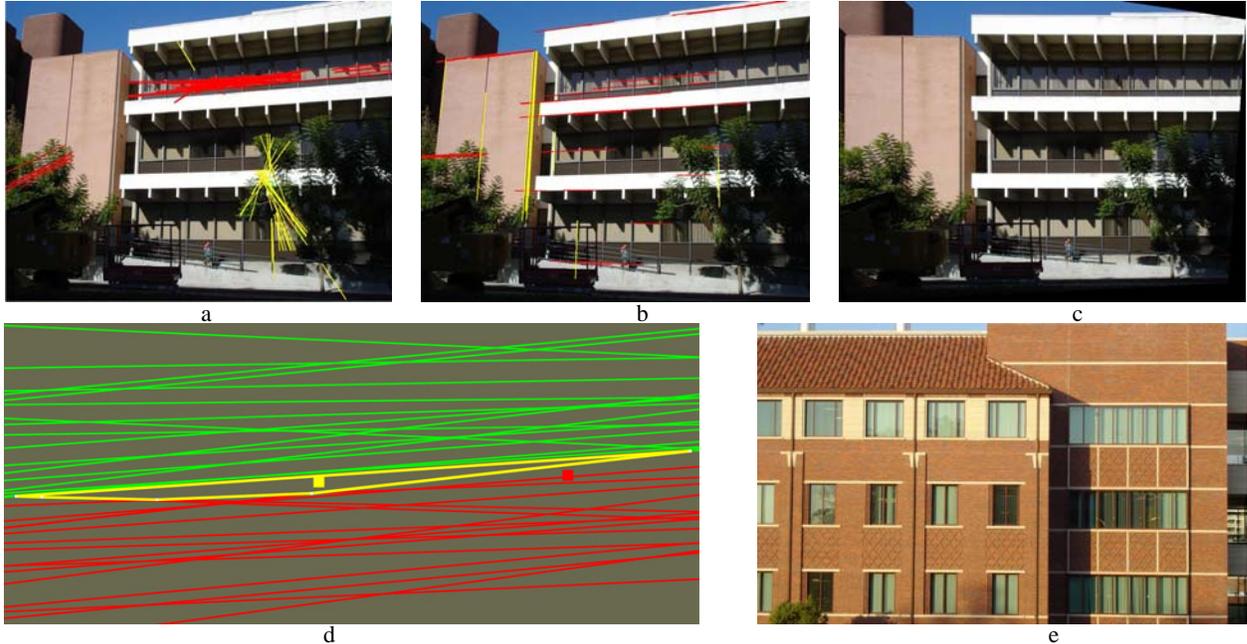


Figure 5. Real image results. A strong perspective image is tested, clustering result before filtering (a) and after filtering (b) are shown, and the image is rectified (c). Figure (d) shows its vanishing hull (yellow edges) and vanishing points estimated using VH method (yellow point) and ML method (red point). A challenging example of weak perspective image is also tested (e).

of the vanishing hull is the optimal solution of the vanishing point, its variance defines the accuracy of the estimation, and its shape determines the stability of the vanishing point.

Future work includes using a more accurate edge error model to reduce the requirement of large number of edges, and apply this method to more real images.

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